

# Project Euler Solutions

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Last updated on 2 Sep 2020

## Contents

<b>1</b>	<b>Multiples of 3 and 5</b>	<b>3</b>
<b>A</b>	<b>About</b>	<b>6</b>
<b>B</b>	<b>Document License</b>	<b>6</b>
<b>C</b>	<b>Code License</b>	<b>7</b>

```
// -5---10---15---20---25---30---35---40---45---50---55---60---65---
#include <iostream.h>

int main()
{
    std::cout << "hello, world\n";
}

// 9
// 10
// 11
// 12
// 13
// 14
// 15
// 16
// 17
// 18
// 19
// 20
// 21
// 22
// 23
// 24
```

# 1 Multiples of 3 and 5

## 1.1 Problem

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

## 1.2 Solution

Let  $S_n = \{x \mid x \text{ is a multiple of } n \text{ and } 0 < x < 1000\}$ .

From the above definition we get,

$$\begin{aligned} S_3 &= \{3, 6, 9, \dots, 993, 996, 999\}, \\ S_5 &= \{5, 10, 15, \dots, 985, 990, 995\}, \\ S_{15} &= \{15, 30, 45, \dots, 960, 975, 990\}. \end{aligned}$$

We want to find

$$\sum_{x \in (S_3 \cup S_5)} x.$$

If we merely add the sum of all the numbers in  $S_3$  to the sum of all numbers in  $S_5$ , then we add the numbers that are multiples of both 3 and 5 twice, that is, we add the numbers that are multiples of 15 twice. Therefore, we must subtract the sum of the multiples of 15 below 1000

to get the required sum, that is,

$$\begin{aligned}\sum_{x \in (S_3 \cup S_5)} &= \sum_{x \in S_3} + \sum_{x \in S_5} - \sum_{x \in (S_3 \cap S_5)} \\ &= \sum_{x \in S_3} + \sum_{x \in S_5} - \sum_{x \in S_{15}}.\end{aligned}$$

The above sum can be written as

$$3 \sum_{k=1}^{\lfloor \frac{m}{3} \rfloor} k + 5 \sum_{k=1}^{\lfloor \frac{m}{5} \rfloor} k - 15 \sum_{k=1}^{\lfloor \frac{m}{15} \rfloor} k$$

where  $m = 999$ . Note that  $\sum_{k=1}^n k$  is the  $n$ th triangular number. Let us denote it as  $T_n$ , that is,

$$T_n = \sum_{k=1}^n k = \frac{n(n+1)}{2}.$$

The required sum can now be written as

$$3T_{\lfloor \frac{m}{3} \rfloor} + 5T_{\lfloor \frac{m}{5} \rfloor} - 15T_{\lfloor \frac{m}{15} \rfloor}$$

where  $m = 999$ . The result is

$$\begin{aligned}&3T_{\lfloor \frac{999}{3} \rfloor} + 5T_{\lfloor \frac{999}{5} \rfloor} - 15T_{\lfloor \frac{999}{15} \rfloor} \\ &= 3T_{333} + 5T_{199} - 15T_{66} \\ &= 3 \cdot \frac{333 \cdot 334}{2} + 5 \cdot \frac{199 \cdot 200}{2} - 15 \cdot \frac{66 \cdot 67}{2} \\ &= 3 \cdot 55611 + 5 \cdot 19900 - 15 \cdot 2211 \\ &= 233168.\end{aligned}$$

## 1.3 Program

```
#include <iostream>

int triangular(int n)
{
    return n * (n + 1) / 2;
}

int main()
{
    int result =    3 * triangular(999 / 3)
                +  5 * triangular(999 / 5)
                - 15 * triangular(999 / 15);
    std::cout << result << '\n';
}
```

## A About

This document contains solutions to Project Euler problems.

This document was last updated on 2 Sep 2020.

The source code for these solutions are available at <https://github.com/susam/lab/tree/master/projecteuler>.

## B Document License

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